Second Level

International Maths Olympiad

by **Maths Olympiad Foundation**

Time : 45 Mins | **Class-XII** | **Maximum Marks : 50**

General Instructions:

1. **All questions are mandatory.** Read the questions carefully before selecting an answer.
2. The question paper contains **25 MCQ**, each carrying 2 **marks**.
3. No negative marking for unattempt questions; one mark will be deducted for each incorrect /overwriting or multiple answers. Illegible answers may not be evaluated.
4. Answers must be marked by writing the correct option **(A/B/C/D)** in the **Answer Sheet**.
5. **Use of calculators, smart watches, or any electronic devices is strictly prohibited.**
6. Rough work must be done **only in the A4 sheet provided by the invigilator**.
7. Ensure that your **Name, Roll Number, School Code, and other required details** are correctly filled in the Answer Sheet before submission.
8. The **final Answer Sheet along with the question paper must be submitted to the invigilator** before leaving the examination hall.
9. Any form of **unfair practice will lead to immediate disqualification**.

Q.1. **Let $f(x) = ax^3 + bx^2 + cx + 41$ be such that $f(1) = 40, f'(1) = 2, f''(1) = 4$, then the value of $a^2 + b^2 + c^2$ is:**

(a) 62 (b) 73 (c) 54 (d) 5

Q.2. **Let $f(x) = \dfrac{\sin x + \cos x - \sqrt{2}}{\sin x - \cos x}$,**

$x \leftarrow [0, \pi] - \left\{\dfrac{\pi}{4}\right\}$ **then** $f\left(\dfrac{7\pi}{12}\right) f'\left(\dfrac{7\pi}{12}\right)$ **is equal to:**

(a) $\dfrac{-2}{3}$ (b) $\dfrac{2}{9}$

(c) $\dfrac{-1}{3\sqrt{3}}$ (d) $\dfrac{-2}{3\sqrt{3}}$

Q.3. **Which of the following statements are correct about $f(x) = \sin x + \frac{1}{2}\cos 2x$ in $\left[0, \frac{\pi}{2}\right]$?**

(i) $f'(x) = \cos x - \sin 2x$

(ii) The critical points of the function are $x = \pi/6$ and $x = \pi/2$

(iii) The minimum value of the function is 2.

(iv) The maximum value of the function is $\frac{3}{4}$

The given options:

(a) (i), (ii) and (iv) only (b) (ii) and (iii) only

(c) (i) (ii), (iii), (iv) (d) (ii), (iii) and (iv) only

Q.4. **The rate of change of the total surface area (in cm^2/S) of a hemi-sphere with respect to radius 'r' at $r = \sqrt[3]{1.331}$ cm is:**

(a) 66π (b) 6.6π (c) 3.3π (d) 4.4π

Q.5. **If $5f(x) + 4f\left(\frac{1}{x}\right) = x^2 - 2 \;\forall\; x \neq 0$ and $y = 9x^2 f(x)$, then y is strictly increasing in:**

(a) $\left(0, \frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$ (b) $\left(\frac{-1}{\sqrt{5}}, 0\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$

(c) $\left(\frac{-1}{\sqrt{5}}, 0\right) \cup \left(0, \frac{1}{\sqrt{5}}\right)$ (d) $\left(-\infty, \frac{-1}{\sqrt{5}}\right) \cup \left(0, \frac{1}{\sqrt{5}}\right)$

Q.6. $\int_0^{\pi/2} \frac{1 - Cot\, x}{Cosec\, x + Cos\, x}$ ***dx* equals**

(a) 0 (b) $\frac{\pi}{4}$ (c) ∞ (d) $\frac{\pi}{12}$

Q.7. $f(x) = \int_0^x (t + \sin(1 - e^t))\, dt, x \,\varepsilon\, R$

then $\lim_{x \to 0} \frac{f(x)}{x^3}$ **is equal to:**

(a) $\frac{1}{6}$ (b) $\frac{-1}{6}$

(c) $\frac{-2}{3}$ (d) $\frac{2}{3}$

Q.8. **The parabola $y^2 = 4x$ divides the area of circle $x^2 + y^2 = 5$ in two parts. The area (in sq. units) of the smaller part is equal to:**

(a) $\frac{2}{3} + 5\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$

(b) $\frac{1}{3} + 5\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$

(c) $\frac{1}{3} + \sqrt{5}\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$

(d) $\frac{2}{3} + \sqrt{5}\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$

Q.9. **The differential equation** $y = xp + \sqrt{x^2 p^3 + y}$

where $p = \frac{dy}{dx}$ **is:**

(i) of order 1 **(ii) of degree 1**

(iii) of order 2 **(iv) of degree 3**

Choose the correct answer from:

(a) (i) and (ii) only

(b) (i) and (iv) only

(c) (i) and (iii) only

(d) (iii) and (iv) only

Q.10. **If m is the degree and n is the order of the given differential equation:**

$$\frac{x^3\left(\frac{d^3y}{dx^3}\right)^2 + 2x^2\left(\frac{d^2y}{dx^2}\right)^3}{(x+1)^5} = \left(3x - \frac{d^2y}{dx^2}\right)^4$$

Then:

(a) $m - n = 2$

(b) $m + n = 5$

(c) $m = 4, n = 3$

(d) $n = 3$ but value of m is not defined

Q.11. $|A| = \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$

Under which of the following conditions will | A | be equal to zero?

(i) $a - 2p = b - 2q = c - 2r = 0$

(ii) $x = y = z = 0$

(iii) $a : b : c = x : y : z$

Choose the correct option from the following:

(a) Only (ii)

(b) Only (i) and (ii)

(c) Only (i) and (iii)

(d) (i), (ii) and (iii)

Q.12. If $A = \frac{1}{\pi}\begin{bmatrix} \sin^{-1}(\pi x) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & \cot^{-1}(\pi x) \end{bmatrix}$ $B = \frac{1}{\pi}\begin{bmatrix} -\cos^{-1}(\pi x) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & -\tan^{-1}(\pi x) \end{bmatrix}$

Then A–B is equal to:

(a) I (b) O (c) 2 I (d) $\frac{1}{2}$I

Q.13. **If the domain of the function $f(x) = \sin^{-1}\left(\frac{x-1}{2x+3}\right)$ is R – (α, β)**
Then 12 αβ is equal to:

(a) 32 (b) 24 (c) 40 (d) 36

Q.14. **If $\text{Cos}^{-1} + \alpha + \text{Cos}^{-1}\beta + \text{Cos}^{-1}\gamma = 3\pi$**
Then $\alpha(\beta + \gamma) + \beta(\gamma + \alpha) + \gamma(\alpha + \beta)$ is equal to:

(a) 0 (b) 1 (c) 6 (d) 12

Q.15. **Which of the following is not an equivalence relation on Z?**

(a) a R b $\Leftrightarrow$ a + b is an even integer
(b) a R b $\Leftrightarrow$ a – b is an even integer
(c) a R b $\Leftrightarrow$ a $\leq$ b
(d) a R b $\Leftrightarrow$ a = b

Q.16. **Match List-I with List-II**

List I		List II	
(A)	$f(x) = \frac{1}{x}, f: \mathbf{R} - \{0\} \to \mathbf{R} - \{0\}$	**(I)**	**Neither injective nor subjective**
(B)	$f(x) = x^2, f: \mathbf{N} \to \mathbf{N}$	**(II)**	**Subjective but not injective**
(C)	$f(x) = x^2, f: \mathbf{R} \to \mathbf{R}$	**(III)**	**Injective but not subjective**
(D)	$f: \{1, 2, 3\} \to \{1, 2\}$ **by** $f = [(1, 1), (2, 2), (3, 1)]$	**(IV)**	**Injective and subjective**

Choose the correct answer from the given options:

(a) A – IV B – I C – II D – III
(b) A – III B – IV C – I D – II
(c) A – II B – III C – IV D – I
(d) A – IV B – III C – I D – II

Q.17. Let R be an equivalence relation on finite set A having n elements $n > 5$. Then R cannot have:

(A) n ordered pairs
(B) $(n - 1)$ ordered pairs
(C) $(n - 2)$ ordered pairs
(D) $(n - 5)$ ordered pairs

Choose the correct option from the following:

(a) (i) and (iii)
(b) (i), (ii) and (iii)
(c) (ii), (iii) and (iv)
(d) (iii) and (iv)

Q.18. If $|\vec{a}| = 3$, $|\vec{b}| = 4$, $|\vec{a} \times \vec{b}| = 10$. Then $|\vec{a} \cdot \vec{b}|^2$ is equal to:

(a) 22
(b) 88
(c) 44
(d) None of these

Q.19. If $\vec{u} = \vec{i}$, $\vec{v} = \vec{j}$ and $\vec{w} = \vec{k}$ are unit vectors. Which of the following is the angle between $(\vec{v} \times \vec{u})$ and $\vec{w}$.

(a) 90°
(b) 180°
(c) 0°
(d) Can not be determined

Q.20. Choose the correct statement if
A (0, 0, 2), B (0, 3, 2), C (5, 3, 2), and D (5, 0, 2) are the vertices of a quadrilateral.

(I) AB is parallel to CD
(II) BC is parallel to AD
(III) AB is perpendicular to BC
(II) ABCD is a rectangle

(a) Only (i) and (ii)
(b) Only (ii) and (iii)
(c) Only (i), (ii) and (iii)
(d) All of the above

Q.21. The shortest distance between the lines l_1 and l_2 given by:

$\hat{r} = a\hat{i} + 2\hat{j} - \hat{k} + \alpha\,(2\hat{i} - \hat{j} + \hat{k})$ **and** $\hat{r} = (\hat{i} - \hat{j} + \hat{k}) + \mu\,(2\hat{i} - \hat{j} + \hat{k})$ **is** $\dfrac{\sqrt{35}}{6}$**, then the value of 'a' can be:**

(a) 0, –8
(b) 0, 8
(c) 2, 6
(d) –2, 6

Q.22. Let α be the angle between the lines whose direction cosines satisfy the equation $l + m - n = 0$ and $l^2 + m^2 - n^2 = 0$

Then the value of $\text{Sin}^4\alpha + \text{Cos}^4\alpha$ is:

(a) $\dfrac{3}{4}$
(b) $\dfrac{5}{8}$
(c) $\dfrac{1}{2}$
(d) $\dfrac{3}{8}$

Q.23. Which of the following cannot be the direction ratios of straight line

$$\frac{x-3}{2} = \frac{2-y}{3} = \frac{z+y}{-1} \ ?$$

(a) 2, –3, –1 (b) –2, 3, 1 (c) 2, 3, –1 (d) 6, –9, –3

Q.24. M & N are two events such that P(M $\cap$ N) = 0. Which of the following is equal to: P | M | M $\cup$ N)

(a) $\frac{P(M)}{P(N)}$ (b) $\frac{P(N)}{P(M) + P(N)}$

(c) $\frac{P(M)}{P(M) + P(N)}$ (d) $\frac{P(M)}{P(M) \times P(N)}$

Q.25. A and B are two events such that P(A) ≠ 0 and P(B/A) = 1, then:

(a) ACB (b) BCA

(c) B = Q (d) None of the above